**324. Proposed by R. D. CARMICHAEL, Princeton University.**

Sum the *finite* series

$$\frac{16n^2-2^2}{4!}-\frac{(16n^2-2^2)(16n^2-4^2)}{6!}+\frac{(16n^2-2^2)(16n^2-4^2)(16n^2-6^2)}{8!}-\ldots$$

where $n$ is a positive integer.

**Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.**

$$\cos m\theta=1-\frac{m^2}{2!}\sin^2\theta+\frac{m^2(m^2-2^2)}{4!}\sin^4\theta-\frac{m^2(m^2-2^2)(m^2-4^2)}{6!}\sin^6\theta+\ldots$$

Let $m=4n$, then

$$\cos 4n\theta=1-\frac{16n^2}{2!}\sin^2\theta+\frac{16n^2(16n^2-2^2)}{4!}\sin^4\theta$$

$$-\frac{16n^2(16n^2-2^2)(16n^2-4^2)}{6!}\sin^6\theta+\ldots$$

Let $\theta=\frac{1}{2}\pi$, then $\cos 4n\theta=\cos 2n\pi=1$.

$$\therefore 1=1-\frac{16n^2}{2!}+\frac{16n^2(16n^2-2^2)}{4!}-\frac{16n^2(16n^2-2^2)(16n^2-4^2)}{6!}$$

$$+\frac{16n^2(16n^2-2^2)(16n^2-4^2)(16n^2-6^2)}{8!}-\ldots$$

$$\therefore \tfrac{1}{2}=\frac{(16n^2-2^2)}{4!}-\frac{(16n^2-2^2)(16n^2-4^2)}{6!}$$

$$+\frac{(16n^2-2^2)(16n^2-4^2)(16n^2-6^2)}{8!}+\ldots$$

## GEOMETRY.

**345. Proposed by LLOYD HOLSINGER, Bradley Polytechnic Institute, Peoria, Ill.**

If a variable polygon move in such a way that its $n$ sides turn severally round $n$ fixed points $O_1$, $O_2$, ..., $O_n$ while $n-1$ of its vertices slide, respectively, along $n-1$ fixed straight lines $v_1$, $v_2$, ..., $v_{n-1}$, then the last vertex will describe a conic; and the locus of the point of intersection of any pair of non-adjacent sides will also be a conic. Cremona's *Projective Geometry*.

Solution by HOWARD C. FEEMSTER, A. B., York College, York, Neb.

Let the sides of the polygon $a_1$, $a_2$, ..., $a_n$ turn severally around the $n$ fixed points $O_1$, $O_2$, ..., $O_n$, describing $n$ flat pencils, $O_1$, $O_2$, ..., $O_n$, while the $n-1$ vertices $A_1$, $A_2$, ..., $A_{n-1}$ slide, respectively, along the $n-1$ fixed straight lines, $v_1$, $v_2$, ..., $v_{n-1}$, forming $n-1$ ranges, $A_1$, $A_2$, ..., $A_{n-1}$. Pencils $O_1$ and $O_2$ are perspective with range $A_1$; ranges $A_1$ and $A_2$ are perspective with pencil $O_2$; pencils $O_2$ and $O_3$ are perspective with range $A_2$, thus making the $n$ pencils and $n-1$ ranges projective.

Now the projective pencils $O_1$ and $O_n$ or any two of the non-consecutive pencils (using above order) will not, in general, be perspective, and every such intersection will thus describe a conic range as its locus. *Encyclopedia Britannica,* "Projective Geometry," Sec. 45.

**347. Proposed by W. J. GREENSTREET, M. A. Marling School, Stroud, England.**

**$ABC$ is a triangle, and $D$, $E$, $F$, are the mid points of the arcs of its nine-point circle cut off by $BC$, $CA$, $AB$, respectively. The inscribed circle touches these sides at $X$, $Y$, $Z$. Are the lines $DX$, $EY$, $FZ$ concurrent? A purely geomatrical discussion required.**

Solution by BENJ. F. FINKEL, Ph. D., Drury College, Springfield, Mo.

In my *Mathematical Solution Book,* page 461, it is shown that the perpendiculars to the chords cut off by the nine-point circle, at the midpoints intersect in a point $S$, which is the center of the nine-point circle. These perpendiculars produced from the chords to the subtended arcs, bisect the smaller arcs in $D$, $E$, and $F$. Since the in-circle is tangent to the chords or chords produced in $X$, $Y$, $Z$, perpendiculars erected at these points meet in a point, $I$, the center of the in-circle. Drawing the lines $DX$ and $EY$, and producing them until they meet $IS$, or $IS$ produced, in $C$ and $C'$, respectively, we have the triangles $SDC$ and $SDC'$.

The triangles $SDC$ and $IXC$ are similar, as are also $SDC'$ and $IYC'$. Hence, $SD$, (the radius of the nine-point circle) : $IX$, (the radius of the in-circle) $=SC : IC$; and similarly, $SE$, the radius of the nine-point circle: $IY$, the radius of the in-circle, $=SC' : IC'$. Hence, $SD-IX : IX=SC-IC(=SI) : IC$, and $SE-IY : IY=SC'-IC'(=SI) : IC'$. Hence, $IC=IC'$ and, therefore, $DX$ and $EY$ intersect the line joining the nine-point circle and the in-circle in the common point $C$. In the same way, it may be shown that the line $FZ$ intersects the same line in the point $C$. Hence, the three lines are concurrent.

## CALCULUS.

**275. Proposed by C. N. SCHMALL, 604 East 5th Street, New York City.**

**Explain fully why the circular measure of an angle is used in the calculus.**